[This paper](http://atarazanas.sci.uma.es/docs/articulos/16692688.pdf) gives the following matrix equation:

Where is the Jacobian transformation from polar to cartesian coordinates, is the partial of the point that a laser scan hits (in cylindrical coordinates) with respect to , is the partial with respect to time, and . Here, represented a range measurement at the angle . The partial derivatives can be approximated as:

Since is already determined using the imu, the matrix equation can be adjusted as:

Solving this equation using least squares involves solving the following matrix equation:

Multiplying the matrices above and expanding the block matrix terms gives the following:

Since only and are desired, solving the matrix requires doing Gaussian elimination on the bottom two rows until you get a 2x2 matrix in the bottom right corner as shown below:

Since the top-left nxn matrix is diagonal and since the entire matrix is symmetric, this can be solved very easily (in O(n) time). The program initializes and to zero. Each time there is a valid range (meaning , and are within the minimum and maximum range), and are incremented by 1 (since there initial values should be ). The and matrices also get subtracted to the corresponding value needed for Gaussian elimination (for example, gets subtracted by ). Furthermore the values in the summations are added to the matrix (since that is there initial value). After going through the entire scan, the x- and y-velocities are calculated by .